

Department of Applied Mathematics and Statistics  
The Johns Hopkins University

SEMINAR

Jim Fill  
Dept. of Applied Mathematics & Statistics  
The Johns Hopkins University

April 12, 2007  
304 Whitehead Hall  
Refreshments: 3:30 p.m.  
Seminar: 4:00 p.m.

A (MINOR) MIRACLE:  
DIAGONALIZATION OF A BOSE–EINSTEIN MARKOV CHAIN

ABSTRACT

Over the past twenty years or so, quite a few techniques have been developed for bounding the mixing times of Markov chains. However, it remains true that the most thorough analysis can be given in the (miraculous) case that the chain’s one-step transition matrix can be explicitly diagonalized. We present one such “miracle” for an ergodic chain whose stationary distribution  $\pi$  is Bose–Einstein configurations (i.e., the uniform distribution over configurations) of  $n$  balls in  $k$  boxes, and another for a generalization of this chain involving the Ewens distribution on permutations. It is quite elementary to sample directly from  $\pi$ , but the chain itself is of interest as an example of the “big-moves” Burnside process, introduced by computer scientists Mark Jerrum and Leslie Goldberg and closely connected with Pólya’s theory of counting, for sampling uniformly from the orbits of a finite set under the action of a finite group.

(This is joint work with Persi Diaconis.)