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SEMINAR

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304 Whitehead Hall
Refreshments: 3:30 p.m.
Seminar: 4:00 p.m.

TURBULENT DIFFUSION OF LINES AND CIRCULATIONS

ABSTRACT

Among the basic principles of fluid dynamics are the theorems on vortex motion of Helmholtz (1858) and the law of conservation of circulations of Kelvin (1869). These results imply that vortex lines are "frozen-in" to an inviscid Eulerian fluid and move as material lines. The classical theorems were derived implicitly assuming that the solutions of the fluid equations remain smooth in the zero-viscosity limit. We present a theorem that Hölder singularities of sufficient strength (exponent $\leq 1/2$) are necessary to violate the Kelvin–Helmholtz Theorem. Such singularities are, however, observed to be present in turbulent fluid flow at high Reynolds number and we present numerical results (with Shiyi Chen) that the Kelvin–Helmholtz Theorem indeed breaks down there. However, this poses a deep theoretical puzzle, since the Kelvin–Helmholtz Theorem is central to our current understanding of the turbulent energy cascade, based on G. I. Taylor's notion of vortex-stretching (1937). We conjecture a "generalized Kelvin Theorem" according to which the fluid circulations—while not strictly conserved in the inviscid limit—are (backward) martingales of a generalized flow. This idea is motivated by recent beautiful work of probabilists Le Jan and Raimond (2002, 2004) on a simplified model of turbulence, in which the fluid velocity is modelled by a stochastic field that is white-noise in time and non-Lipschitz (but Hölder continuous) in space. We shall describe the key results of LeJan and Raimond on "stochastic splitting" of material points and "anomalous dissipation" of passive scalars. We then point out a formal generalization to material lines and fluid circulations.