Applied Mathematics I (500.303)
Extra Credit Projects

The following rules apply:

- Each project is worth 30 homework points or 15 in-class exam points. You must specify which of these options you'd prefer. If no option is specified when the project is turned in, the points will be applied to the homework score.
- The deadlines (see below) are firm. No extensions!
- All projects must be completed **INDIVIDUALLY**. You may however consult with the instructor if you have questions. No points will be awarded if there is evidence of group work.
- You may consult the course text and any notes handed out during the course during FALL 2001. All other materials are prohibited.
- You must include a cover page, signed and dated, with the following statement in your own handwriting:

> "I certify that the material submitted is entirely my own work. I have adhered to all of the rules outlined for the extra credit projects."

Project	**Due Date**
I	October 31, 2001
I	November 21, 2001
III	December 12, 2001

PROJECT 1 (Due October 31, 2001)

Note: It may be helpful to read Example 6 in Section 2.13 in your book.

The static deflection $y(x)$ of a uniform straight beam of length L carrying a load $w(x)$ per unit length is found from the fourth-order differential equation

$$EIy'''' = w(x),$$

where E is Young's modulus of elasticity and I is the moment of inertia of a cross section of the beam.

1. For a cantilever beam clamped at its left end ($x = 0$) and free at its right end ($x = L$), $y(x)$ must satisfy

$$y(0) = 0, \quad y'(0) = 0, \quad y''(L) = 0, \quad y'''(L) = 0.$$

The first two conditions state that the deflection and slope are zero at $x = 0$, and the last two conditions state that the bending moment and shear force are zero at $x = L$.

Use the Laplace transform to solve when a constant load w_0 is uniformly distributed along the length of the beam. That is $w(x) = w_0 \quad 0 < x < L$.

Hint: Let $c_1 = y''(0)$ and $c_2 = y'''(0)$. Use the conditions at $x(L)$ to find c_1 and c_2.
Note: To apply the Laplace Transform, which utilizes an integration from 0 to ∞, we tacility assume that $w(x)$ and $y(x)$ are defined on $0 < x < \infty$ rather than $0 < x < L$.

2. Solve the previous problem, again using Laplace transforms, when

$$\begin{aligned} w(x) &= 0 && 0 < x < L/3 \\ &= w_0 && L/3 < x < 2L/3 \\ &= 0 && 2L/3 < x < L. \end{aligned}$$

3. Suppose the beam carries a concentrated load P_0 at $x = L/2$. Then

$$w(x) = P_0 \delta(x - L/2).$$

Solve using Laplace transforms.

4. Suppose the beam carries a concentrated load P_0 at $x = L/2$ as in problem 3. But now the beam is clamped at both ends so the conditions that must be satisfied are:

$$y(0) = 0, \quad y'(0) = 0, \quad y(L) = 0, \quad y'(L) = 0.$$

Solve using Laplace transforms.

PROJECT 2 (Due November 21, 2001)

1. Kreyszig problem 3.5.15
2. Kreyszig problem 3.5.16

3. Let $x(t)$ be a harmful insect population (e.g. aphids) that under natural conditions is held somewhat in check by $y(t)$, a benign predator insect population (e.g. ladybugs). Assume that $x(t)$ and $y(t)$ satisfy the basic predator-prey equations:

$$\begin{aligned} dx/dt &= ax - bxy \\ dy/dt &= -cy + dxy \end{aligned}$$

$$a,\ b,\ c,\ d > 0 \text{ (constants).}$$

Now, suppose that an insecticide is employed that kills (per unit time) the same fraction $f < a$ of each species of insect.

Find the critical points of this new system, and determine the type and stability of each one. Explain whether the use of the insecticide was productive or not.

PROJECT 3 (Due December 12, 2001)
Kreyszig problem 11.4.20